empirical large-signal HEMT model. The principal intrinsic elements, C_{gs}, C_{gd}, g_m, and g_{ds}, are represented as functions of the gate and drain bias voltages.

We characterized an AlGaAs/GaAs HEMT with a gate 0.3 μm long and 100 μm wide with our large-signal model. We included the model in a commercially available circuit simulator as a user-defined element and designed a 30/60-GHz frequency doubler operating at $V_{gs} = -0.55$ V and $V_{ds} = 2.0$ V. The fabricated doubler had a conversion loss of 5 dB with a 30-GHz 0-dBm input signal. The experimental data agreed well with the calculations.

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Efficient Method for Scattering from a Homogeneous, Circular, Cylindrical Shell with an Inhomogeneous Angular-Region

S. Jegannathan

Abstract— **The two-dimensional (2-D) scalar problem of a circular, dielectric, cylindrical shell exposed to transverse magnetic (TM) incident field is considered. The shell is considered to be homogeneous everywhere, except in a narrow angular-region where it is allowed to be inhomogeneous. The problem is formulated using the moment method (MM). It is shown that the resulting system of MM equations could be very efficiently solved employing a new theory of diagonally-perturbed circulant matrices. The method presented here could be applied for thin shells as well as shells which are "not-so-thin." Results of computer simulations are also provided verifying the validity of the method proposed.**

I. Introduction

The scattering behavior of various dielectric bodies may be analyzed employing the moment method (MM) [1]. The main limitation of the MM is that it requires the solution of a large, nonsparse system of linear equations. The method demands significant computing resources when implemented directly. It is therefore important to recognize and use any structure present in the coefficient matrix.

In the present work, we consider scattering from a circular, dielectric, cylindrical shell. The shell is considered to be homogeneous everywhere, except in a narrow angular-region where it may be inhomogeneous. We show that the resulting system of MM equations could be efficiently solved employing the new theory of diagonally-perturbed circulant matrices [5] developed in the following sections. An alternative to the approach presented here is to employ the combination of the conjugate-gradient method (CGM) and the FFT [2]–[4]. For the scatterer under consideration, we show that the method presented here works faster than the CGM-FFT technique.

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II. Circulants

Consider the $N \times N$ circulant matrix $\mathbf{\Pi}_0$ given by

$$\mathbf{\Pi}_0 = \begin{pmatrix} c_0 & c_{N-1} & c_{N-2} & \cdots & c_1 \\ c_1 & c_0 & c_{N-1} & \cdots & c_2 \\ c_2 & c_1 & c_0 & \cdots & c_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{N-1} & c_{N-2} & c_{N-3} & \cdots & c_0 \end{pmatrix}. \quad (1)$$

The elements of the above matrix may be complex; it is assumed that they satisfy

$$c_r = c_{N-r} \qquad \text{for} \quad r = 1, 2, \cdots, N-1. \quad (2)$$

Let $\mathbf{\Pi}_1, \mathbf{\Pi}_2, \cdots, \mathbf{\Pi}_L$ be matrices such that they are the same as matrix $\mathbf{\Pi}_0$ but with its m_1^{th}, $(m_1, m_2)^{th}$, $\cdots$,$(m_1, m_2, \cdots, m_L)^{th}$, respectively, main diagonal element(s) perturbed by (δ_1), (δ_1, δ_2),$\cdots$, $(\delta_1, \delta_2, \cdots, \delta_L)$, respectively, where m_i, $i = 1, 2, \cdots, L$ are integers such that $1 \leq m_1, m_2, \cdots, m_L \leq N$, $\delta_1, \delta_2, \cdots, \delta_L$ are arbitrary complex constants; $1 \leq L \leq N$ and $m_i \neq m_j$ when $i \neq j$. Assuming the appropriate inverses exist, we now have

$$\mathbf{\Pi}_n^{-1} = \mathbf{\Pi}_{n-1}^{-1} - \tilde{\delta}_n \mathbf{v}_{n-1} \mathbf{v}_{n-1}^T \qquad \text{for} \quad n = 1, 2, \cdots, L \quad (3)$$

where

$$\tilde{\delta}_n = \frac{\delta_n}{1 + \delta_n c'_{n-1}}. \quad (4)$$

In the above equations $\mathbf{v}_{n-1}$ and c'_{n-1} are the m_n^{th} column, and the m_n^{th} diagonal element, respectively, of $\mathbf{\Pi}_{n-1}^{-1}$. Equation (3) can be derived as follows: For $n = 1, 2, \cdots, L$, we have

$$\mathbf{\Pi}_n = \mathbf{\Pi}_{n-1} + diag(0 \ \cdots \ \delta_n \ \ 0 \ \cdots \ 0) \quad (5)$$

where in the diagonal matrix of the above equation, δ_n occupies the appropriate m_n^{th} diagonal position. Equation (5) may be written as

$$\mathbf{\Pi}_n = \mathbf{\Pi}_{n-1} + \mathbf{u}_n \mathbf{u}_n^T \quad (6)$$

where $\mathbf{u}_n$ is the $N \times 1$ column vector, all the elements of which are zero except the m_n^{th} element; the m_n^{th} element of $\mathbf{u}_n$ equals $\delta_n^{\frac{1}{2}}$. Now, we have from the matrix inversion lemma [6]

$$\mathbf{\Pi}_n^{-1} = \mathbf{\Pi}_{n-1}^{-1} - \frac{\mathbf{\Pi}_{n-1}^{-1} \mathbf{u}_n \mathbf{u}_n^T \mathbf{\Pi}_{n-1}^{-1}}{1 + \mathbf{u}_n^T \mathbf{\Pi}_{n-1}^{-1} \mathbf{u}_n}. \quad (7)$$

Evidently

$$\mathbf{\Pi}_{n-1}^{-1} \mathbf{u}_n = \sqrt{\delta}_n \mathbf{v}_{n-1}. \quad (8)$$

From (2) and (5), for $n = 1, 2, \cdots, L$ we have

$$\left(\mathbf{\Pi}_{n-1}^{-1}\right)^T = \mathbf{\Pi}_{n-1}^{-1}. \quad (9)$$

Transposing both sides of (8), and substituting (9), we get

$$\mathbf{u}_n^T \mathbf{\Pi}_{n-1}^{-1} = \sqrt{\delta}_n (\mathbf{v}_{n-1})^T \quad (10)$$

from which we have

$$\mathbf{u}_n^T \mathbf{\Pi}_{n-1}^{-1} \mathbf{u}_n = \delta_n c'_{n-1}. \quad (11)$$

Substituting (8), (10), and (11) in (7), we obtain

$$\mathbf{\Pi}_n^{-1} = \mathbf{\Pi}_{n-1}^{-1} - \frac{\delta_n \mathbf{v}_{n-1} \mathbf{v}_{n-1}^T}{1 + \delta_n c'_{n-1}}. \quad (12)$$

Substituting (4) in (12), we obtain (3). From (3) it follows that for an integer L satisfying $1 \leq L \leq N$, we have

$$\mathbf{\Pi}_L^{-1} = \mathbf{\Pi}_0^{-1} - \sum_{r=1}^{L} \tilde{\delta}_r \mathbf{v}_{r-1} \mathbf{v}_{\mathbf{r-1}}^{\mathbf{T}}. \quad (13)$$

Let us now consider solving a system of equations of the form $\mathbf{\Pi}_L \mathbf{x} = \mathbf{y}$. Using (13) we may write

$$\mathbf{x} = \mathbf{\Pi}_0^{-1} \mathbf{y} - \sum_{r=1}^{L} \tilde{\delta}_r \mathbf{v}_{r-1} \mathbf{v}_{r-1}^T \mathbf{y}. \quad (14)$$

Let us for the present assume that $\mathbf{\Pi}_L^{-1}$ is available in the decomposed form of (13); that is, $\mathbf{\Pi}_0^{-1}$, the $\tilde{\delta}$, and $\mathbf{v}$ are all known. If this is the case, the following algorithm may be employed to evaluate $\mathbf{x}$ (14) efficiently.

A. Algorithm A

Since $\mathbf{\Pi}_0$ is a circulant matrix, so also is $\mathbf{\Pi}_0^{-1}$ Hence

1) Evaluate $\mathbf{\Pi}_0^{-1}\mathbf{y}$. Call this result $\mathbf{x}'$.
   For $r = 1, 2, \cdots, L$, repeat steps ii) –iv).
2) Evaluate $\mathbf{v}_{r-1}^T \mathbf{y}$ The result is a scalar.
3) Multiply $\tilde{\delta}_r$ with the result of the above step.
4) Multiply the result of the previous step and the column vector $\mathbf{v}_{r-1}$ and subtract the result from $\mathbf{x}'$. Replace $\mathbf{x}'$ with this result.

The result $\mathbf{x}'$ after L iterations is the required solution $\mathbf{x}$ as seen from (14). The computational merits of Algorithm A as well as the following Algorithm B, which is to be used for decomposing $\mathbf{\Pi}_L^{-1}$ in the required form, are discussed in §IV.

Let us consider (14). The vectors $\mathbf{v_0}, \mathbf{v_1}, \cdots, \mathbf{v_{L-1}}$ are the $m_1^{th}, m_2^{th}, \cdots, m_L^{th}$ columns of $\mathbf{\Pi}_0^{-1}, \mathbf{\Pi}_1^{-1}, \cdots, \mathbf{\Pi}_{L-1}^{-1}$, respectively. The constants $\tilde{\delta}_n$ are as defined in (4), we see that these constants depend on c'_{n-1}, besides on the known δ_n. The quantity c'_{n-1}, as defined earlier, is the m_n^{th} diagonal element of $\mathbf{\Pi}_{n-1}^{-1}$; that is, c'_{n-1} is the m_n^{th} element of $\mathbf{v}_{n-1}$. In view of this, we need to calculate $\mathbf{\Pi}_0^{-1}$ and the $m_2^{th}, m_3^{th}, \cdots, m_L^{th}$ columns of $\mathbf{\Pi}_1^{-1}, \mathbf{\Pi}_2^{-1}, \cdots, \mathbf{\Pi}_{L-1}^{-1}$, respectively, to decompose $\mathbf{\Pi}_L^{-1}$; a full description of the latter matrices will only be redundant. Exploiting this, we may propose the following efficient algorithm to decompose $\mathbf{\Pi}_L^{-1}$.

B. Algorithm B

1) Invert the circulant matrix $\mathbf{\Pi}_0$.
2) The m_1^{th} column of $\mathbf{\Pi}_0^{-1}$ is $\mathbf{v}_0$. c'_0 is the m_1^{th} element of $\mathbf{v}_0$. $\tilde{\delta}_1$ may be evaluated now from (4).
   If $L = 1$, we have already got the required decomposition. However, if L is greater than unity, the following steps must be executed:
3) For $r = 2, 3, \cdots, L$ repeat.
1) Set $\mathbf{v}_{r-1} = m_r^{th}$ column of $\mathbf{\Pi}_0^{-1}$.
2) For $i = 1, 2, \cdots, r-1$ repeat
   a) Evaluate $\delta' = \tilde{\delta}_i (\mathbf{v}_{i-1})_{m_r}$ where $(\mathbf{v}_{i-1})_{m_r}$ denotes the m_r^{th} element of $\mathbf{v}_{i-1}$.
   b) Set $(\mathbf{v}_{r-1})_j = (\mathbf{v}_{r-1})_j - \delta' (\mathbf{v}_{i-1})_j$ where $(.)_j$ denotes the j^{th} element of the column vector; do this for all the values of j, i.e., $j = 1, 2, \cdots, N$.

The above algorithms can be used even when all the N diagonal elements of the circulant are perturbed; however, computational advantages are derived only when the number of perturbed elements L is much smaller than N, as shown in §IV.

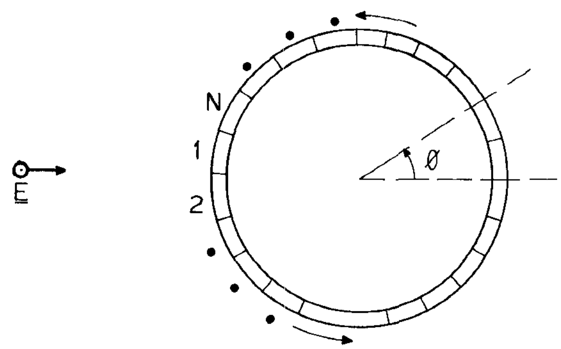

Fig. 1. A shell is divided into N cells.

III. Scattering from a Circular, Cylindrical Shell

Fig. 1 depicts the cross section of a shell divided into cells as required by the MM [1]. Implementation of the MM yields a system of linear equations of the form

$$(\mathbf{I} + \mathbf{K}\epsilon)\mathbf{e} = \mathbf{e}^{\boldsymbol{i}} \quad (15)$$

where $\mathbf{I}$, $\mathbf{K}$, ϵ are all $N \times N$ matrices, $\mathbf{e}$ and $\mathbf{e}^{\boldsymbol{i}}$ are $N \times 1$ column vectors. In the above equation, $\mathbf{I}$ is the unity matrix, ϵ is a diagonal matrix given by $\tilde{\epsilon} - \mathbf{I}$ where the n^{th} diagonal element of $\tilde{\epsilon}$ is the same as the dielectric constant of the cell n, $\mathbf{K}$ is a matrix whose element on the m^{th} row and n^{th} column depends upon the distance between the centres of cells m and n; the n^{th} element of $\mathbf{e}$ and $\mathbf{e}^{\boldsymbol{i}}$ are the same as the the total and incident field intensities at the centre of cell n.

Once (15) is solved for $\epsilon\mathbf{e}$, the scattered field can be determined at any point in space by means of a simple calculation as described in [1]. It is evident that the task reduces to solving a circulant system, if the shell is homogeneous; if it is inhomogeneous, we have to deal with a diagonally-perturbed circulant. Moreover, when the cell division is performed in a way such that the centres of the cells are angularly-equispaced, the condition specified by (2) is evidently met. In view of this, Algorithms A and B could be straightaway applied to solve (15).

Even if the shell is "not-so-thin" requiring a few radial divisions as well, Algorithms A and B could be applied to solve the resultant MM system; in that case, we would be dealing with á block-circulant matrix [5] instead of a circulant.

IV. Computational Complexity

The computational complexity of Algorithms A and B can be determined as follows: step 1) of Algorithm A requires the evaluation of N-point FFT thrice [7]. Besides, it also requires N arithmetic operations to be performed. Each execution of steps 2)–4) requires $(2N-1) + (1) + (2N) = 4N$ arithmetic operations. Hence, Algorithm A requires on the whole $N(4L+1)$ arithmetic operations and the evaluation of N-point FFT thrice. Similarly, it can be shown that Algorithm B requires roughly $\frac{(L^2-L+2)}{2}N$ arithmetic operations, besides the execution of two N-point FFT's. These figures would be somewhat different, if intermediate results are stored and reused. In

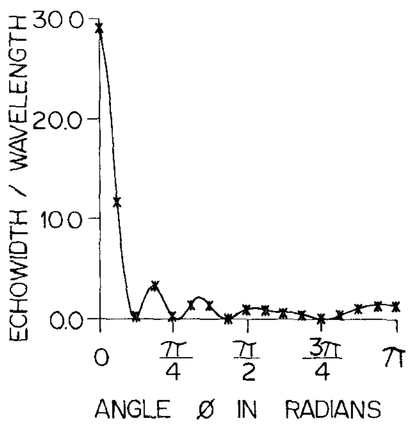

Fig. 2. Distant scattering pattern of a homogeneous cylindrical shell with an inhomogeneous patch —— Method presented here * * * * Gaussian elimination method dielectric constant = $6.0 + j$ when $\phi = -\frac{7\pi}{128}$ to $\frac{7\pi}{128}$; = 3.0 elsewhere; mean radius = 1.0 λ, thickness = 0.05 λ number of cells = 128.

any case, when N is large and $L << N$, the overall computational effort is proportional to $N \log N$.

In comparison, the CGM-FFT method is $O(N^2 \log N)$, the Gaussian elimination method is $O(N^3)$. Thus, when N is large and $L << N$, the method presented here is significantly faster.

V. Numerical Results

The MM equations of the shell described in Fig. 2 were solved employing the Gaussian elimination method, the CGM-FFT method and the method presented here. The execution time for the three methods were 77550 msec, 9110 msec, and 561 msec, respectively. This includes the time for calculating the coefficients of the matrix. These results were obtained with Fortran 77 compiler on Toshiba T4400C notebook. Fig. 2 depicts the distant scattering pattern calculated from the solution of the MM equations. The exact agreement obtained verifies the correctness of the method proposed.

As a second example, a shell of mean radius 4.0λ and of the same thickness as the previous example was considered. The dielectric constant of the shell was taken to be 3.0 except in the region $-\frac{9\pi}{128} \leq \phi \leq \frac{9\pi}{128}$ where it was taken to be $6.0 + j$; the angle ϕ is as indicated in Fig. 1. The size of the MM matrix was 512 × 512; in the previous example it was 128 × 128. In view of the large size of the matrix, only the CGM-FFT method and the method presented here were implemented. The execution time for the two methods were 70530 msec and 3350 msec, respectively. Thus, in both the cases, the method presented here was considerably faster.

VI. Conclusion

In this paper, we have presented a method for solving the MM equations of a circular, dielectric, cylindrical shell efficiently. Although we focused our attention on a homogeneous shell with a narrow inhomogeneous angular-region, the theory derived here does not assume that the perturbations should be contiguous; the method works even when there are more than one inhomogeneous patch. As the shell is a useful model for scatterers such as co-axial cables, volcanic pipes, blood vessels and arteries, efficient determination of the scattering pattern will be of use in applications such as crack detection.

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Conductor-Loss Limited Stripline Resonator and Filters

Chen-Yu Chi and Gabriel M. Rebeiz

***Abstract*— We report on stripline resonators on thin dielectric membranes that show dispersion-free, conductor-loss limited performance at 13.5 GHz, 27.3 GHz, and 39.6 GHz. The unloaded-Q (Q_u) of the resonators increases as $\sqrt{f}$ with frequency and is measured to be 386 at 27 GHz. The measured results agree well with a new conformal mapping analysis. The stripline resonators are used in a micromachined state-of-the-art planar interdigitated bandpass filter at K-band frequencies. Excellent agreement has been achieved between the microwave model at 850 MHz and the 20 GHz filter. The micromachined filter exhibits a passband return loss better than −15 dB and a conductor-loss limited 1.7 dB port-to-port insertion loss (including input/output CPW line loss) at 20.3 GHz.**

I. Introduction

Silicon micromachined technology has been used recently to build low-loss lumped elements, K-band and W-band filters, resonators and couplers [1]–[3]. Micromachined components, suspended on thin dielectric membranes, do not suffer from dielectric and dispersion loss up to terahertz frequencies [4]. Also, with the advantage of the monolithic microwave/millimeter-wave integrated cirucits (MMIC) fabrication process, batch fabricated micromachined components can have identical responses for use in large volume satellite receiver systems and future personal communication systems at microwave and millimeter-wave frequencies.

In this paper, we report on the performance of stripline and microstrip resonators on thin dielectric membranes at microwave and millimeter-wave frequencies. The stripline results are shown to be conductor-loss limited but the microstrip results show a small free-space radiation loss component at 13 GHz and this radiation loss

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